Selection and the Role of Small Business Owners in Firm Dynamics

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Abstract

What makes small businesses small? Standard theories attribute size differences to efficiency, access to credit, or some combination of the two. I present new evidence on persistent earnings differentials of business owners relative otherwise similar wage and salary workers that are difficult to explain with these two sources of heterogeneity. I develop and evaluate a Hopenhayn (1992) style model of firm heterogeneity with an occupational choice, and I introduce varying preferences for business ownership consistent with the non pecuniary motives documented in Hurst and Pugsley (2011). The model nests both efficiency- and credit- based accounts of firm heterogeneity, and it allows me to isolate the role of each mechanism. I show how ignoring preference heterogeneity leads to an overestimate of the tightness of borrowing constraints and a miscalibration of the productivity process, in particular for small businesses. Finally, I evaluate an implication of the model when extended to sectors with varying fixed costs. Low fixed cost sectors will have a greater concentration of taste-driven business owners and thus larger earnings differentials. Empirically, I find evidence for this result when I compare earnings differentials in the services and manufacturing sectors.

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1 Introduction

Small businesses range from corner stores to high tech startups. In the United States, firms with fewer than 20 employees account for almost 90 percent of all firms and employ 20 percent of the workforce. What explains why these firms are small? In this paper, I show that the standard sources of heterogeneity in models of firm dynamics leave some key features of small firms unexplained, and I propose and evaluate a model with an alternative source of heterogeneity. The paper makes three contributions. First, I document patterns of persistent compensating differentials between earnings of business owners and workers. These differentials create difficulties for the standard theories of firm dynamics. Second, I develop an alternative model that includes ex-ante heterogeneity in the preferences of households towards running a business. I call this a “taste” for entrepreneurship. The model with tastes not surprisingly is better able to explain the earnings differentials. More importantly, incorporating tastes introduces a selection on tastes mechanism that turns out to be particularly relevant for understanding small firms. To better compare preference heterogeneity with alternative sources of heterogeneity, the model nests simplified versions of two popular models of firm dynamics that derive from heterogeneity in productivity and in access to credit. Finally, using the model, I show that preference heterogeneity accounts for over 40 percent of small firms. I show how ignoring tastes can overstate the role of credit constraints and efficiency in understanding small firms.

Modern theories of firm dynamics build on two main sources of heterogeneity to explain the distribution of firm size. In the first, firm size is a function of exogenous differences in efficiency. The Hopenhayn (1992) model of selection and its derivatives are prime examples. In these models, firms have decreasing returns to scale and some form of fixed cost that together imply an optimal scale. This fixed cost could be technological as in the original Hopenhayn (1992) model, or it could follow from an occupational choice; running the firm requires time that could otherwise be allocated to the labor market. In efficiency based models, the distribution of firm size follows entirely from the underlying distribution of productivity. Larger scale firms have lower costs than small firms.

In the second kind of model, differences in firm size follow from the wealth distribution of households. In these models, financial market imperfections, such as imperfect enforceability of contracts, require the firm post collateral in order to borrow capital. The credit constraints

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1 Although I focus on productivity and credit frictions, there are many other potential sources of differences among firms. Rossi-Hansberg and Wright (2007) consider differences in the accumulation of an industry specific form of human capital as a factor of production. See Luttmer (2010) for an excellent summary of models of firm dynamics.

2 The Jovanovic (1982) model of learning is also an example. Size is still driven by efficiency, but a firm owner must learn about the parameters of her cost function.
prevent firms from borrowing enough capital to immediately reach their efficient scales. Wealthier entrepreneurs are able to run larger firms because they have the resources to borrow larger amounts of capital. In Cagetti and De Nardi (2006), for example, all entrepreneurs are equally efficient, and the distribution of firm size follows entirely from the distribution of wealth.

These types of models, often in combination, have had enormous success in explaining firm entry, exit and growth, as well as explaining the savings patterns of business owners. Cooley and Quadrini (2001) combine both sources of heterogeneity to explain age and size dependence in firm growth. Moll (2010), Buera, Kaboski, and Shin (2011), and Midrigan and Xu (2010) have all used the combination of heterogeneity in productivity and credit availability to measure the extent of capital misallocation when highly productive entrepreneurs cannot borrow or are deterred from starting a business. Collectively, the implication of these models is that firms are either small because they are less productive or because they are more productive but slowly growing into their efficient scale.

When combined with an occupational choice decision, these models imply selection solely on comparative advantage. Entrepreneurs choose to run businesses because of a higher expected rate of return on their labor and financial capital. This is harder to support empirically. I present evidence that business earnings are in fact almost 30 percent lower than comparable worker earnings for the typical business owner. Although, previous work by Hamilton (2000) and Moskowitz and Vissing-Jørgensen (2002) has documented low returns from business ownership overall, I examine the heterogeneity underlying the aggregate returns. I show these earnings differences are driven largely by owners of small firms. Moreover, they persist throughout the firms’ life cycle, even as uncertainty over firm performance fades. Heterogeneity in efficiency or access to credit cannot explain these facts.

To confront these earnings differentials I develop a model that introduces preference heterogeneity towards running a business. The model constructs a Hopenhayn and Rogerson (1993) style general equilibrium and nests versions of efficiency- and credit- heterogeneity driven firm dynamics. People vary in their tastes for running a business, which are fixed forever. Tastes enter preferences as a discrete jump in utility from running a business.

Given a set of skills that evolve stochastically, people make an occupational choice each period to either run a business or work for a business run by another household. All businesses produce the same good with a decreasing returns to scale technology. A limited span of control along with the outside option of employment imply an optimal scale for each firm. In the absence of credit frictions, solely the ability of the business owner would determine the size of the firm. However, I include an exogenous collateral constraint that limits the debt the firm can take on and by extension limits the size of the firm. Credit constrained
firms grow into their optimal scale.

The key contribution of the model is to provide a role for preferences in the occupational choice decision. In the stationary equilibrium business owners are driven by comparative advantage and tastes. Incorporating preferences changes the composition of operating businesses by lowering the minimum wealth and ability levels needed to run a business. As a result, a set of businesses operate that would otherwise not have formed. All of the marginal firms are small scale businesses, which are small because their size is already the optimal scale. These businesses exist alongside higher ability but credit constrained firms growing into their optimal scale, resembling the heterogeneity among small businesses in the United States.

The marginal business owners will have lower pecuniary earnings than comparable workers in return for the non pecuniary compensation generated by their preferences. Since marginal businesses are smaller scale, they are less likely to be credit constrained. Whereas credit constrained firms have high expected returns to match their higher borrowing costs, the expected return of marginal firms is close to the risk free rate.

In a calibrated version of the model that matches the patterns of earnings differentials, I find that taste-driven marginal firms account for over 40 percent of all firms. Although previous work has acknowledged the possibility of non pecuniary compensation of business owners, this paper is the first of its kind to quantify the role of tastes in the composition of small businesses. By letting tastes interact with the standard forms of heterogeneity, which we know to be extremely important, I show the consequences of ignoring tastes. This addresses an important challenge with integrating an occupational choice into models of firm dynamics.

Without selection on tastes, we need another way to entice proprietors of small firms to stay in business in spite of more lucrative employment earnings. Purely credit constraint explanations require these small firms to be constrained with high expected returns. To achieve this with only productivity requires a stochastic process that makes earnings only temporarily low. However, in both cases, low earning small scale firms cannot survive in the long run when there is a viable outside option.

The change to the selection mechanism depends in part on tastes that do not scale with the size of the business. Preferences shift the occupational choice margin without affecting the intensive margin. This comes at some cost by breaking the homogeneity in model and complicating the solution. However, tastes independent of size function well in accounting for the earnings differentials of small business owners.

As a final evaluation of the model and this form of preference heterogeneity, I consider an extension to two sectors with different levels of fixed costs so that each sector has a different
average scale. Suppose business owners are equally capable of running firms in either sector given their ability. The implication of selection on size-independent tastes is that the sector with lower fixed costs will contain a higher proportion of taste-driven business owners, and thus the earnings differentials relative to working will be larger. I find this same pattern empirically. Earnings differentials in the manufacturing sector with larger fixed costs are much smaller than the service sector with smaller fixed costs.

The paper proceeds as follows. In Section 2 I describe the empirical support for compensating differentials of business owners relative to workers. I show these differences are persistent and unlikely to follow from alternative explanations. In Section 3 I develop the model of firm dynamics that incorporates preference heterogeneity. Section 4 describes the results of the model calibrated to explain the observed differences in earnings. Section 5 describes a two sector extension of the model that evaluate against evidence on earnings differences that vary by sector to support the assumed form of preference heterogeneity. Section 6 concludes.

2 Tastes for Entrepreneurship

Empirically, preference heterogeneity presents as compensating differentials in the choice between employment and business ownership. A worker with a preference for running a business is willing to accept lower earnings in return for the non pecuniary compensation of running the business. In this section, I document large and persistent earnings differences between business owners and workers to support this claim. There is already a large literature examining the returns to business ownership. Recently, Hamilton (2000) compares earnings profiles of workers and business owners, and Moskowitz and Vissing-Jørgensen (2002) compare the return on capital held in private equity relative to a diversified portfolio of public equity. These studies find that the typical business owner could achieve higher income and bear less idiosyncratic risk by allocating her time to employment and capital to a diversified equity portfolio. Non pecuniary compensation arising from a preference for business ownership explains these findings. I go beyond the previous literature by disaggregating the distributions of earnings and I uncover a pattern of earnings differences that makes the preferences for business ownership an even more compelling explanation.

2.1 Data Description

To measure business and worker earnings, I combine the 1996, 2001, and 2004 panels of the Survey of Income and Program Participation (SIPP). This SIPP is a household level
survey conducted by the US Census Bureau. Each panel consists of approximately 40,000 households who are interviewed every four months for a total of 9 to 12 waves, or 3 to 4 calendar years. Combining the three panels provides a large number of observations on earnings from 1996 through 2007. Similar to the Consumer Population Survey (CPS), there are overlapping rotation groups so that in each calendar month one fourth of the households are surveyed. Unlike the CPS, the SIPP was specifically designed to give comprehensive measures of income from employment, business ownership, and household financial assets. Participating households answer questions about their income for each of the four months within the wave. Moreover, supplemental questions about stock variables, such as business assets and the household balance sheet, are timed so that households give measurements exactly one year apart.

I limit the sample to working-age male participants to limit the influence of a labor force participation decision. The sample includes all males, aged 18 to 65, who either work as an employee or own and operate as business as a full time activity. In each month I classify a person as either a worker or business owner according to his income sources. In cases where both types of income are present, I use reported hours to break the tie. I exclude casual businesses with annual earnings less than $2500 or farm income. I aggregate the monthly observations into annual earnings measures of the longest employment or business ownership spell during the year, and I exclude spells of less than 12 weeks to exclude seasonal employment or businesses. The final sample includes 198,492 annual observations from 1996 to 2007 with 175,378 from workers and 22,754 from business owners.

Table 1 describes the sample across each SIPP panel. Business owners are slightly older, more likely to be married, and much less likely to be African American than workers. Educational attainment is higher for business owners, who are more likely to hold college or graduate degrees. These demographics closely match similar samples from the PSID and CPS. The business ownership rate is about 11 percent, which is comparable to the CPS. Its decline in the most recent panel is consistent with a decline in the self employment rate measured in the CPS over the last decade. Table 2 shows the age distribution of businesses owned by business owners in the sample. It is very close to the age distribution of all employer firms measured in the Business Dynamic Statistics (BDS) for 2005, which is compiled from administrative data by the US Census Bureau.

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3I define full time as working at least 35 hours in a typical week.
4The data appendix gives a full description of the sample restrictions and additional information about the SIPP.
5See for example Hurst, Li, and Pugsley (2010).
6See for example Fairlie (2011); Hipple (2010).
2.2 Measuring Compensating Differentials

The distributions of worker and business earnings in the SIPP are noticeably different. Figure 1 plots estimated kernel densities of the conditional distributions of weekly earnings for workers and for business owners. In order to make individuals as comparable as possible, I first condition out demographic controls and observable measures of human capital. Then I estimate the density of the residuals, which I re-center at the unconditional average weekly earnings. The typical business owner is earning less than the typical worker. Median business earnings are about 30 percent lower than worker earnings. The distribution of business earnings is also characterized by a fatter tail, although it is hard to discern in the picture. By the 75th percentile, business earnings overtake worker earnings and are almost 100 percent higher in the top 5 percent. Although I focus on weekly earnings in order to be consistent with the inelastic the labor supply in the model, the differences are even more pronounced after adjusting for hours worked. These differentials mirror earlier work by Hamilton (2000) who, using the 1984 SIPP panel, finds that median business owners earn up to 35 percent less than comparable workers over 10 years. These earnings differences on their own do not establish a compensating differential. There are a number of stories compatible with the differences in earnings. I show that alternative explanations are less likely.

Selection The most obvious confounding factor is selection on ability. The business owners could have been even worse workers. Although it is impossible to rule this out definitively, selection is unlikely to account for the differences. The short-dashed line in Figure 1, is an estimated earnings distribution on the subsample of workers who will will have started a business in a future SIPP wave. If business owners were even worse workers, we would expect their earnings as workers prior to starting a business to reflect their lower ability. Instead, the distribution of worker earnings for future business owners looks very close to the overall distribution of worker earnings. Selection is still an important factor, both empirically and in the model. However, selection on ability alone does not explain the earnings gaps. Rather, I show that the occupational choice is driven by selection on tastes and ability, with tastes playing the leading role in these earnings differences.

7I control for race, marital status, work limiting disability, and whether the person has ever retired from a job. To measure human capital, I use educational attainment dummies, and quadratics for business or job tenure and years of potential labor market experience.

8Table 1 shows that business owners work about 12 percent more hours per week on average than workers. After adjusting for hours, the differences in median earnings rises to 37 percent. The gap in earnings actually narrows in the tail where high earning workers and business owners both work similarly long hours.

9The 1984 SIPP panel is 3 years; he constructs synthetic cohorts using quadratics for experience and job tenure.
Expectations  Business owners could also have noisy signals or even overoptimistic beliefs about their own abilities. In both cases, over time less productive business owners should learn their true type and eventually return to employment if employment still offers a better outside option. However, the earnings differences are extremely persistent. Business owners earning less than comparable workers continue to survive and fail to close the gap in earnings. Table 3 summarizes the results of a series of earnings regressions. I run average weekly earnings on a set of controls, human capital measures, time effects, and a set of dummies indicating business ownership, business size, and business age. Small firms have fewer than 25 employees. Panel A shows both quantiles and averages of the distribution of earnings differences for younger businesses open for fewer than 10 years. The median business owner earns about $240 less per week than a comparable worker. This is 27 percent lower than the median worker’s weekly earnings of $891. Even the 75th percentile small business owner earns $90 less than a comparable worker, which is still 8 percent lower than the 75th percentile worker’s earnings of $1166. For larger business owners, the exact opposite is true. Throughout the distribution, they do better than comparable workers.

It may not be surprising that owners of larger firms do so well if high ability drives business size. However, even after ten years of successfully running businesses, the small business owners have made almost no progress closing the earnings gap, whereas larger firms have widened the gaps even further. Median earnings differences of small firms decrease by less than four dollars. Earnings differences driven by uncertainty should diminish over time as the less successful business owners return to higher paying employment. Instead, the earnings difference are nearly unchanged. Many businesses do exit over time. Within the BDS, almost 60 percent of new firms exit within the first 5 years of operation. Yet, among those businesses that survive, there is a large set of small businesses who survive in spite of lower earnings.

It is possible low performing business owners continue operating because the value of the outside employment option diminishes over time. To address this possibility, in Figure 2, I plot estimated conditional earnings distributions of two subsamples. The dashed line is the distribution of earnings of businesses that have operated at least 5 years. The solid line is the distribution of earnings of workers at a new job. The distribution of earnings for workers at a new job dominates the distribution of business earnings over all but the right tail. The outside option still looks attractive for most business owners, given employment earnings comparable workers.
Measurement Error  Finally, earnings differences could reflect measurement error. It is well known that small business owners underreport their business income to tax authorities.\textsuperscript{10} Unfortunately, this measurement error extends to household surveys. In Hurst, Li, and Pugsley (2010), we estimate that the self employed underreport from 20 to 25 percent of their income in the Consumer Expenditure Survey and Panel Study of Income Dynamics. Since SIPP participants report income every 4 months instead of annually, underreporting is costlier than copying reported income from the participants’ income tax records.

However, even if the SIPP shared the same extent of underreporting, this source of measurement error does not resolve the earnings differences. First, employment often includes additional non-income compensation, such as employer sponsored health care coverage. Previous work by Hamilton (2000) shows that only 7 percent of full time employees purchase their own health insurance, and that the self employed are 23 percent less likely than employees to have a policy issued in their own name. The non-wage fringe benefits provided by employers will partially offset the underreporting. Second, business income is riskier than employment income. Risk averse people are willing to trade off some income for small income fluctuations. Even if true business earnings were equivalent to worker earnings, the risk adjusted value of business earnings would still be lower than worker earnings.\textsuperscript{11}

Preference Heterogeneity  Preferences for owning a business would explain persistence in the earnings differences. Independent sources of data also validate this claim. In Hurst and Pugsley (2011), using data from the Panel Study of Entrepreneurial Dynamics we examine a panel of 1,214 individuals who are just about to start a business in the first wave. The survey asks participants for reasons why they are starting the business, as well asks them to rank particular motives by importance. We classify the free form answers into categories such as new product idea, income motives, and non pecuniary reasons (be my own boss, flexible schedule, like the work, and so on), and we find that non pecuniary reasons are equally as important as other more traditional motives for starting a business. Additionally, we find that the presence of non pecuniary motives forecasted lower ex-ante expectations of employment growth relative to income and new product motives. Collectively, the pattern of persistent earnings differences between business owners and workers, as well as the explicit accounts of new entrepreneurs, indicate preference heterogeneity is a first order consideration in the occupational choice decision. In the following section, I integrate varying tastes into

\textsuperscript{10}See Slemrod (1985) or Andreoni, Erard, and Feinstein (1998) for example.

\textsuperscript{11}Another source of measurement error is the difference between business profit and the cash drawn from the business. Businesses, especially when credit constrained, will reinvest earnings into the business. In the robustness appendix, I consider alternative measures of business income that more closely align with business profits. These measures show the same persistence in earnings differentials.
a model of firm dynamics to show the role that tastes play in shaping the dynamics of firms.

3 A Model of Firm Dynamics with Tastes

In this section I develop an equilibrium model where households select business ownership on both profitability and tastes to accord with the persistent earnings differences I document above. Selection on tastes changes the composition of business owners in the distribution of firms. People who would otherwise choose higher earnings in employment are now willing to run a business. In the stationary distribution, small scale and lower performing mom and pops will exist alongside credit constrained and growing firms with high expected returns. The model expands on recent work by Midrigan and Xu (2010) and incorporates preference heterogeneity with an occupational choice into a general equilibrium environment.

3.1 Preliminaries

There are a large number of long-lived households that make up a unit measure. Household \( i \in [0, 1] \) has preferences over a single final good ordered by

\[
E \left[ \sum_{t=0}^{\infty} \beta^t \left( \log C_{it} + v_i K_{it} > 0 \right) \right].
\]

The household discounts future consumption at rate \( 1/\beta - 1 \). The innovation to preferences is the parameter \( v_i \), which captures a “taste” for entrepreneurship. Importantly, it is independent of the marginal utility of consumption. The parameter \( \nu \) simply changes the level of utility when the household runs a business, i.e., when \( K_{it} > 0 \). Although, I assume that \( \nu \geq 0 \), this is a convenient normalization. Tastes \( \nu \) describe a relative preference for running a business, and could equivalently be expressed as distaste for employment. Taste \( \nu_i \) is heterogeneous in the population of households, and its value is fixed forever.

Tastes are distributed over households with distribution function \( G(\nu) \). Households further differ by a set of skills composed of stochastic entrepreneurial ability \( Z_{it} \) and labor efficiency \( Y_{it} \).

Each household possesses one unit of labor supplied inelastically to either employment or business ownership. I assume, as in Lucas (1978), that managing a household-owned business is a full time activity. The household labor supply is indivisible and allocated to exactly one of the two activities: \( j \in \{W, B\} \). \( W \) refers to working, and \( B \) refers to running a business.
Business Ownership

Household-run businesses produce the final good using capital and labor supplied in efficiency units by the working households. Business-owning households control a decreasing returns technology

\[ Z_{it}^{1-\eta} \left( K_{it}^{\theta} L_{it}^{1-\theta} \right)^{\eta} \]

that depends on the ability \( Z \) and produces output given the inputs of capital \( K \) and labor \( L \). Labor is measured in efficiency units. The span of control parameter \( \eta \) controls the degree of decreasing returns, as in Lucas (1978).\(^{12}\) Similar to Hopenhayn (1992) style models, households with a large \( Z \) have a larger span of control and are able to run larger scale firms. I choose the final good as the numeraire so that output has price 1. Labor is hired at price \( w_t \) per efficiency unit. Capital is freely converted from output, so its price is fixed at 1, and it depreciates at rate \( \delta \). There are no adjustment costs for capital. Diminishing returns along with the outside option of returning to employment imply an optimal scale for the business that depends only on \( Z_{it} \). However, the actual choices of \( K_{it} \) and \( L_{it} \) may be limited by an exogenous credit constraint.

Credit Constraint

To impose a simple credit friction, I closely follow Midrigan and Xu (2010) and Buera, Kaboski, and Shin (2011) and include intra-period business debt subject to a collateral constraint.\(^{13}\) The idea is that the business is required to fund its outlays on wages and capital \( w_t L_{it} + K_{it} \) at the beginning of the period. If the beginning of period households assets \( A_{it} \) are insufficient to fund the outlays, the household may use the debt market to borrow at risk free rate \( r_f^t \). However, this debt must be repaid at the end of the period.

Figure 3 gives an example of a typical period. The firm starts the period with assets \( A_{it} \). It uses the debt market to fund its outlays on labor and capital \( w_t L_{it} + K_{it} \).\(^{14}\) Then it earns revenues \( Z_{it}^{1-\eta} \left( K_{it}^{\theta} L_{it}^{1-\theta} \right)^{\eta} \), it sells the remaining capital earning \( (1 - \delta) K_{it} \), and it earns the interest on its asset holdings \( r_f^t A_{it} \). Using this intra-period formulation the end of period resources available for consumption or saving are

\[ Z_{it}^{1-\eta} \left( K_{it}^{\theta} L_{it}^{1-\theta} \right)^{\eta} + (1 - \delta) K_{it} - (w_t L_{it} + K_{it}) \left( 1 + r_f^t \right) + \left( 1 + r_f^t \right) A_{it} . \]

\(^{12}\)Decreasing returns could alternatively follow from a monopolistic producer with elastic demand.

\(^{13}\)This form of collateral constraint was first used by Evans and Jovanovic (1989).

\(^{14}\)It is convenient to think of the firm as using the debt market to fund its outlays and collecting the interest on its assets. Since the borrowing rate and the rate of return on assets are identical, this is equivalent to borrowing \( W_{it} L_{it} + K_{it} - A_{it} \) and repaying \( \left( 1 + r_f^t \right) (W_{it} L_{it} + K_{it} - A_{it}) \).
Less asset holdings, business income can be written as

\[ Z_{it}^{1-\eta} \left( K_{it}^{\eta} L_{it}^{1-\theta} \right)^{\eta} - \left( r_{t}^{f} + \delta \right) K_{it} - \left( 1 + r_{t}^{f} \right) w_{t} L_{it} . \]  

(1)

The amount borrowed in the debt market however is limited by a collateral constraint. Business borrowing \( w_{t} L_{it} + K_{it} - A_{it} \) cannot exceed \((\lambda - 1)\) times beginning of period assets \( A_{it} \). The factor \((\lambda - 1)\) is exogenous and constant across all households. With this formulation of the credit constraint, choices of \( L_{it} \) and \( K_{it} \) are limited by

\[ w_{t} L_{it} + K_{it} \leq \lambda A_{it} . \]

(2)

This formulation of the exogenous collateral constraint is convenient since it spans frictionless markets where \( \lambda \to \infty \) and no debt markets with \( \lambda \to 1 \). Although I treat the collateral constraint as exogenous, a similar constraint would arise endogenously in a model where the business owner can run away with the borrowed capital.\(^\text{15}\)

Suppose \( \mu \) is the multiplier on the collateral constraint (2). As suggested by Midrigan and Xu (2010), we can think of the business as if it were facing a borrowing rate of \( r_{t}^{b} = r_{t}^{f} + \mu \). The multiplier acts as an external finance premium.

Since there are no adjustment costs to capital, we can think of the firm as if facing factor prices \( w_{t} (1 + r_{t}^{f}) \) and \( (r_{t}^{b} + \delta) \) for labor and capital respectively in a rental market. Firms will be credit constrained, i.e, \( r_{t}^{b} > r_{t}^{f} \), if the ratio \( A_{it}/Z_{it} \) is less than \( b_{t}^{*} \) where

\[ b_{t}^{*} = \frac{1}{\lambda^{\eta} \eta^{\frac{1}{1-\eta}}} \left( \frac{\alpha}{1 - \alpha} \frac{w_{t} (1 + r)}{r_{t}^{f} + \delta} \right)^{\frac{\eta \alpha}{\eta}} \left( \frac{1 - \alpha}{w_{t} (1 + r_{t}^{f})} \right)^{\frac{1-\eta}{\eta}} \left( \frac{w_{t} (1 + r_{t}^{f})}{r_{t}^{f} + \delta} \right) \frac{\alpha}{1 - \alpha} . \]

The threshold is constant when \( w_{t} \) and \( r_{t}^{f} \) are fixed. Increasing \( A_{it} \) or decreasing \( Z_{it} \) moves the ratio \( A_{it}/Z_{it} \) closer to the threshold, which relaxes the collateral constraint and lowers the external finance premium.

**Budgets and Idiosyncratic Risk**

Household’s income depends on the occupational choice. If the household chooses to work, \( j = W \), it earns wage \( Y_{it} w_{it} \); if it chooses to run a businesses, \( j = B \), it receives the net income \( \Pi (A_{it}, Z_{it}) \) from the business. I define business income \( \Pi (A_{it}, Z_{it}) \) as the maximum of (1) subject to (2). Because of the credit constraint, business income depends additionally

\(^{15}\)See Buera, Kaboski, and Shin (2011) Cagetti and De Nardi (2006) for example. In these models the constraint is a non linear function of assets.
on $A_{it}$.

Skills given by ability $Z_{it}$ and labor efficiency $Y_{it}$ are uncertain and their logs follow the AR(1)s

$$\log Z_{it} = a_z + \rho_z \log Z_{it-1} + \sigma_z \varepsilon_t$$

and

$$\log Y_{it} = a_y + \rho_y \log Y_{it-1} + \sigma_y \zeta_t$$

with iid shocks

$$\left( \begin{array}{c} \varepsilon_t \\ \zeta_t \end{array} \right) \sim N(0, I)$$

Although households face idiosyncratic changes to their skills, there are no aggregate shocks, and there are enough households to ensure that a law of large numbers eliminates aggregate uncertainty. The households can only partially insure the idiosyncratic risk by trading in a safe asset that earns risk-free return $r^f_t$. I let $A_{it}$ represent household $i$’s holding of the safe asset at the beginning of period $t$. After production or working the household has access to resources $Y_{it}w_t + \left(1 + r^f_t\right)A_{it}$ if $j = W$ or $\Pi(A_{it}, Z_{it}) + \left(1 + r^f_t\right)A_{it}$ if $j = B$ to allocate to consumption $C_{it}$ or risk free savings $A_{it+1}$.

### 3.2 Recursive Representation

I formulate the household’s problem recursively before looking for a stationary equilibrium. Let $V_i(A, Z, Y)$ be the value of a household who enters the period with assets $A$ and skill vector $(Z, Y)$. Since there is no aggregate uncertainty, I consider a stationary environment where the wage $w$ and risk free rate $r^f$ will be constant and the value function depends on $t$ only through the state variables. The value function is, however, indexed by $i$ since household’s will vary in the size of their entrepreneurial taste draw $\nu_i$.

At the beginning of the period the household decides whether to work or to operate a business. I let $V_i^W(A, Z, Y)$ be the value of a household who chooses to work for that period, and I let $V_i^B(A, Z, Y)$ be the value of a household who chooses to run a business for that period. There is no cost to switching occupations so that

$$V_i(A, Z, Y) = \max_{j \in \{W, B\}} \{V_i^W, V_i^B\}.$$

This formulation of the occupational choice closely follows Cagetti and De Nardi (2006).
The function $V^W$ satisfies the Bellman equation

$$V^W_i (A, Z, Y) = \max_{C, A'} \frac{C^{1-\gamma} - 1}{1 - \gamma} + \frac{1}{1 + \rho} E_{Z,Y} [V_i (A', Z', Y')] \tag{4}$$

subject to the worker’s budget constraint

$$C + A' = (1 + r^f) A + wY .$$

The function $V^B$ satisfies the Bellman equation

$$V^B_i (A, Z, Y) = \max_{C, A'} \frac{C^{1-\gamma} - 1}{1 - \gamma} + \nu_i + \frac{1}{1 + \rho} E_{Z,Y} [V_i (A', Z', Y')] \tag{5}$$

subject to the business owner’s budget constraint

$$C + A' = (1 + r^f) A + \Pi (A, Z)$$

where $\Pi (A, Z)$ incorporates the credit constraint (2).

### 3.3 Recursive Stationary Equilibrium

I look for a recursive stationary competitive equilibrium in this economy in which the overall distribution of households is constant. Although the aggregate state is constant, households within the model accumulate or spend assets and start or shut down businesses as they respond to idiosyncratic shocks, similar to a Bewley (1977) style model.

Given a unit measure of households, with tastes distributed according to distribution function $G (\nu)$, and transition function $F (Z, Y|Z, Y)$ for the evolution of skills $Z$ and $Y$, a recursive stationary equilibrium consists of constant prices $W$ and $r^f$ and the following objects

1. Policy functions $J_\nu (A, Z, Y) \in \{W, B\}$ and $A'_\nu (A, Z, Y)$, indexed by the value $\nu$, which solve the Bellman equations (3)-(5)

2. Policy functions $K (A, Z, Y)$ and $L (A, Z, Y)$ that maximize profits (1) subject to the credit constraint (2).

3. An invariant distribution $\psi^*_\nu (A, Z, Y)$ indexed by $\nu$ that satisfies

$$\psi^*_\nu (A, Z, Y) = \int_{A'_\nu (A, Z, Y) \in A} F (Z, Y|Z, Y) \psi^*_\nu (dA, dZ, dY)$$
4. And an invariant aggregate distribution over all households

\[ \Psi^* (A, Z, Y) = \int \psi^*_\nu (A, Z, Y) \, dG (\nu) \]

such that all markets clear.

I clear the labor and asset markets to determine prices \( w \) and \( r^f \). Clearing the labor market requires adding up the the labor demands by all business owner households, measured in efficiency units, and ensuring it equals the efficiency units of labor supplied by the remaining worker households

\[
\int \int_{J (A, Z, Y) = B} L (A, Z, Y) \, \psi^*_\nu (dA, dZ, dY) \, dG (\nu) = \int \int_{J (A, Z, Y) = W} Y \psi^*_\nu (dA, dZ, dY) \, dG (\nu)
\]

Although the labor demand is independent of tastes, the occupational choice decision \( J (A, Z, Y) \) does depend on tastes, so I must sum over the labor demands for the business owners of each taste. Similarly, clearing the asset market requires that the savings of both worker and business owner households add up to the total physical capital demands

\[
\int \int A' (A, Z, Y) \, \psi^*_\nu (dA, dZ, dY) \, dG (\nu) = \int \int_{J (A, Z, Y) = B} K (A, Z, Y) \, \psi^*_\nu (dA, dZ, dY) \, dG (\nu)
\]

4 Understanding the Role of Tastes

In the stationary equilibrium tastes play a key role in determining the composition of businesses. Because of their owners’ tastes, small scale businesses start and survive despite higher earnings opportunities as workers. In this section I focus on the role of tastes and use the model to compare the competing roles of credit constraints and productivity. Later, I calibrate the model with and without preference heterogeneity, and I find when calibrated to match the differences in median earnings that tastes account for over 40 percent of the distribution of firms. This is an important force and I describe how ignoring tastes changes the calibration of other productivity and credit parameters.

4.1 Selection in the Model

The model delivers a prominent role for tastes through the selection mechanism. By shifting the thresholds where households are just willing to enter business ownership, tastes \( \nu \) change
the composition of the firm distribution. If we were able to observe $\nu$ and group firms by their owners’ preferences, we would see that firms owned by households with high $\nu$ would be smaller on average and less likely to be constrained.

**Proposition 1.** The average household firm size, measured by sales $Y$, employment $L$, or capital $K$, is decreasing in $\nu$.

The smaller size follows immediately from the change in entry threshold. Suppose $\nu = 0$ and there is a set of households running businesses. As $\nu$ increases, all of the households that were already running businesses are inframarginal. The marginal firms drawn in by smaller ability and wealth thresholds are small scale firms, lowering the overall average size.

**Proposition 2.** The average household external finance premium, measured by $r_t^b - r_f$, is decreasing in $\nu$.

Since the marginal firms are technologically smaller scale, the credit constraint is relaxed or even slack. Without a binding credit constraint there is no external finance premium, and the equilibrium return on capital is equal to the risk free rate. There is no within period uncertainty over the marginal product of capital, so we would not expect to see a risk premium. In the model, higher within period returns are due to the external finance premium imposed by binding credit constraints.

To show how tastes change the selection mechanism, in Figure 4 I plot an example of the occupational choice decision rule plotted over the state space. In the left hand panel, the solid line represents the occupational choice decision rule for households with $\nu = 0$ in an economy with a tighter credit market (low $\lambda$). Households above the line run businesses and households below the line are workers. The decision rule is for a particular value of $Y$ that determines potential employment earnings. It asymptotes vertically as $A$ reaches the minimum level needed to run a business. If there were no uncertainty in $Y$, then this threshold would represent a poverty trap, as in Buera (2009). These poor households would spend down their remaining assets and forever remain workers. With uncertainty in $Y$, even with $\nu = 0$ there is always some probability of reaching a state where it is more profitable to run a business. The decision rule asymptotes horizontally towards the ability level $Z$ where the value of an unconstrained business at that scale is just equal to the value of remaining a worker. The broken line represents a loosening of the credit markets. As $\lambda$ increases, households with sufficiently high levels of $Z$ switch to business ownership. It is important to keep in mind that this is only the participation margin. Households who enter business ownership after relaxing credit markets will still be constrained after they enter. In the model relaxing credit constraints switches higher ability, but poor, households into business ownership.
The right hand panel shows the effect of $\nu > 0$. The solid line again represents the occupational choice rule for $\nu = 0$ for a given $Y$. The broken line shows how the rule for the same value of $Y$ changes as $\nu$ increases. Whereas relaxing credit moves higher ability households with larger scale firms into business, increasing $\nu$ moves lower ability households with small scale firms into business. What discourages these households from running firms is that the businesses are not a large enough scale to be a profitable alternative to employment. Introducing preference heterogeneity provides enough non pecuniary compensation to offset the lower pecuniary returns relative to employment. The key reference point is the value of the outside option. These are still profitable firms, however without $\nu > 0$, the business profits are not large enough to compensate the household for the loss of employment earnings.

These marginal firms are difficult to explain without some form of non pecuniary compensation. In models with credit frictions small firms have high expected returns. This is true in both Cagetti and De Nardi (2006) and Cooley and Quadrini (2001), where all new firms are high ability but face higher borrowing costs than the larger incumbents. With preference heterogeneity, new firms will come from high and low ability households, and only the high ability businesses will be constrained. In this paper’s model, productivity heterogeneity still drives the distribution of firm scales. Without an occupational choice decision, Hopenhayn (1992) type models have no difficulty explaining small firms. However, with an occupational choice, less productive business-owning households return to employment. Raising the value of the outside option truncates the support of sustainable firms. Midrigan and Xu (2010) confront this same problem in their model of misallocation. They adjust the distribution of abilities, so that low ability levels needed to support small firms are only temporary. In all of these cases, however, small firms will eventually either grow or exit. The introduction of preference heterogeneity allows naturally small scale firms to persist.

4.2 Calibrated Results

To quantify the significance of preference heterogeneity, I consider a calibrated version of the model. My goal is to stay close to the existing literature in order to compare the role of tastes with other forms of heterogeneity. To do this, I use the calibration in Midrigan and Xu (2010) as a starting point and describe the adjustments needed to integrate tastes. I solve the model numerically using an endogenous grid method adjusted to accommodate the discrete occupational choice.\textsuperscript{16} This procedure builds on recent work by Fella (2011), and I include the details in Appendix C.

\textsuperscript{16}The discrete choice introduces a non-concavity to the value function. This complicates the standard approach to the endogenous grid method since in the neighborhood of the non-concavity, the first order condition is no longer sufficient for an optimum.
In the calibrated models, I consider the simplest form of preference heterogeneity. A fraction $p$ of households have tastes $v_i = \bar{v}$, and the remaining fraction $(1 - p)$ have $v_i = 0$. I fix $p = 0.4$ which accords with survey evidence from the Kauffman Foundation on the proportion of people who aspire to run a business someday.\textsuperscript{17} I set $\bar{v}$ to match the differences in median earnings of 30 percent between workers and business owners.

Table 4 specifies the benchmark calibration. I use a period length of one year and take as fixed capital elasticity $\alpha = 0.33$, depreciation $\delta = 0.06$, and span of control $\eta = 0.85$. I set $\beta = 0.92$, which will imply an equilibrium risk free rate in the range of 4 to 5 percent. These parameter choices match Midrigan and Xu (2010) and are relatively standard in the literature. For the worker income process $Y$, I use the same specification as Cagetti and De Nardi (2006), which sets $\rho_y = 0.95$ and $\sigma_y = 0.24$ to match a Gini coefficient of 0.38 in the distribution of employment earnings. In the baseline model, I set $\lambda = 50$, which implies relatively well functioning credit markets. This leaves parameters $\rho_z, \sigma_z,$ and $\bar{v}$. I set these parameters to match the business ownership rate, the tail of the distribution of firms, and the difference in median earnings across occupations.

In the baseline case, calibrated to match the earnings differences, tastes play an economically significant role. Nearly half of all firms are due to their owners’ preferences. These firms would not have operated if their decision to enter and exit was driven only by ability and wealth. These marginal firms are small scale firms by definition and are less likely to face credit constraints. Moreover the value of non pecuniary compensation is large. I compute the average value of non pecuniary compensation for the marginal firms by averaging over the marginal utility of consumption for this group to price $\bar{v}$. It represents 114 percent of the average wage. The results also show that selection on ability is still an important force, and they caution against interpreting the empirical median earnings differences as an unbiased estimate. I compute the counterfactual earnings differences for each household, which we cannot observe empirically. Even though median business earnings are 30 percent below median employment earnings, the median business owner earns 14 percent more than the median employee. Among the households with $v_i = \bar{v}$, however, the median business owner makes half of what she could earn as an employee.

Next I shut down preference heterogeneity, but leave the other technological parameters intact. In Model 2 from the table, I leave $\lambda = 50$ and in Model 3, I tighten credit markets by setting $\lambda = 2$. While both models match the distribution of firms, neither comes close to generating the earnings differentials we see empirically.

In Table 6 I look at the implications for the equilibrium distribution of wealth in each of the three models. Only model 3 with tight credit markets comes close to matching the tail of the wealth distribution. Interestingly, preference heterogeneity and tight credit markets have similar effects. A model with preferences and intermediate credit constraints is also able to fit the empirical wealth distribution. This is because both induce a correlation between wealth and business ownership. Because tastes make business ownership a normal good, the wealthy are more likely to own a business, independent of any credit constraints. Credit constraints of course make wealthy business ownership more likely by loosening the credit constraint with increases in wealth. Although both induce a correlation, the selection mechanism differs. As evident in Figure 4, relaxing the credit market pulls out higher ability but poor households into entrepreneurship. Increasing tastes, pulls lower ability but wealthy households into entrepreneurship. The latter is consistent with Hurst and Lusardi (2004) who find that probability of starting a business does not vary with wealth until wealth reaches a high level.

5 Testing the Implications of a Two Sector Extension

The selection mechanism characterized above depends on preference heterogeneity affecting only the occupational choice and not the the intensive margin of continuing firm investment. In this section I test that assumption by evaluating its implications for earnings differentials in a two sector extension to the model. Suppose there are two sectors, each with different per period fixed costs. The additional fixed cost raises the minimum efficient scale so that the average scale will be larger in the sector with the larger fixed costs. The marginal (taste-driven) business owners will enter the sector with the smallest fixed costs, and thus we expect to see larger wage differentials in smaller scale industries. Here I extend the model to show this mechanism, and then show evidence for similar patterns empirically when I compare the manufacturing and services sectors.

5.1 Technologies

There are two sectors. The service sector produces good $S$, and the manufacturing sector produces good $M$. Both are combined into a final good by the CES technology

$$C = \left( \omega S^{\frac{\sigma-1}{\sigma}} + (1 - \omega) M^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma$ represents the elasticity of substitution between the two inputs, and $\omega$ represents the factor share of the service good. The manufacturing good $M$ is also used as physical
capital for both sectors. The market final good is competitive and determines prices $p_S$ and $p_M$. Cost minimizing implies that the price of the final good is

$$P = \left( (1 - \omega) \left( \frac{p_S}{1 - \omega} \right)^{1-\sigma} + \omega \left( \frac{p_M}{\omega} \right)^{1-\sigma} \right)^{1/\sigma}.$$  

I continue to use the final good as the numeraire so that $P = 1$.

The service sector technology remains the same as the one sector model. The manufacturing technology, however, incorporates a fixed cost $\kappa$ denominated in units of the manufacturing good

$$Z^{1-\eta} (K^\theta L^{1-\theta})^\eta - \kappa.$$  

### 5.2 Occupational Choice with Two Sectors

This additional sector introduces a new option to the occupational choice so that the value of household $i$ now becomes

$$V_i(A, Z, Y) = \max_{j \in \{W, S, M\}} \{V_i^W, V_i^S, V_i^M\}. \quad (6)$$  

While, the Bellman equation 3 determining the value of a worker is unchanged, the value of a business now differs by sector. The value of a business in sector $k \in \{S, M\}$ satisfies

$$V^k_i = \max_{C, A' \geq 0} \log C + \nu_i + \beta E \left[ V_i(A', Z', Y') | Z, Y \right]$$

subject to the budget constraint

$$C + A' = (1 + r_f) A + \Pi_k (A, Z).$$  

Business income $\Pi_k (A, Z)$ will again depend on $A$ through a credit constraint. To determine $\Pi_S (A, Z)$, service sector firms maximize

$$p_S Z^{1-\eta} (K^\theta L^{1-\theta})^\eta - (r_f + \delta) p_M K - (1 + r_f) wL \quad (7)$$

subject to the collateral constraint

$$w_i L_{it} + p_M K_{it} \leq \lambda A_{it}. \quad (8)$$
To determine $\Pi_M(A, Z)$ manufacturing firms maximize

$$p_M Z^{1-\eta} (K^\theta L^{1-\theta})^\eta - (1 + r^f) p_M \kappa - (r^f + \delta) p_M K - (1 + r^f) w L$$

subject to the collateral constraint

$$w_t L_{it} + p_M K_{it} + p_M \kappa \leq \lambda A_{it}.$$  \hspace{1cm} (10)

5.3 Two Sector Recursive Stationary Equilibrium

The characterization of the stationary equilibrium changes only slightly. Occupational choice is now one of three possible values, and markets for the service sector and manufacturing sector goods determine prices $p_S$ and $p_M$.

Given a unit measure of households, with tastes distributed according to distribution function $G(\nu)$, and transition function $F(Z, Y|Z, Y)$ for the evolution of skills $Z$ and $Y$, a two sector recursive stationary equilibrium consists of constant prices $p_S, p_M, W$ and $r^f$ and the following objects

1. Policy functions $J_{\nu}(A, Z, Y) \in \{W, S, M\}$ and $A'_{\nu}(A, Z, Y)$, indexed by the value $\nu$, which solve the Bellman equations (3)-(5)

2. Policy functions $K(A, Z, Y)$ and $L(A, Z, Y)$ that maximize profits in the appropriate sector.

3. An invariant distribution $\psi^*_\nu(A, Z, Y)$ indexed by $\nu$ that satisfies

$$\psi^*_\nu(A, Z, Y) = \int_{A'_{\nu}(A, Z, Y) \in A} F(Z, Y|Z, Y) \psi^*_\nu(dA, dZ, dY)$$

4. And an invariant aggregate distribution over all households

$$\Psi^*(A, Z, Y) = \int \psi^*_\nu(A, Z, Y) dG(\nu)$$

such that all markets clear.

Labor markets clear as before. To clear the remaining markets, I introduce several summary measures. First, I use $K$ to represent the aggregate quantity of capital

$$K \equiv \int \int_{J_{\nu}(A, Z, Y) \in \{S, M\}} K(A, Z, Y) \psi^*_\nu(dA, dZ, dY) dG(\nu).$$

21
Then I let $S$ and $M$ represent the total service sector and manufacturing sector output, which I measure as

$$
S \equiv \int \int_{J_{\nu}(A,Z,Y) = S} Z^{1-\eta} (K (A, Z, Y)^{\alpha} L (A, Z, Y)^{1-\alpha})^\eta \psi^*_\nu (dA, dZ, dY) dG (\nu)
$$

and

$$
M \equiv \int \int_{J_{\nu}(A,Z,Y) = M} (Z^{1-\eta} (K (A, Z, Y)^{\alpha} L (A, Z, Y)^{1-\alpha})^\eta - \kappa) \psi^*_\nu (dA, dZ, dY) dG (\nu).
$$

I use $Y \equiv (\omega S^{\frac{\sigma-1}{\sigma}} + (1 - \omega) M^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$ for total output. Using this notation, asset markets clear by

$$
\int \int A_{\nu}^t (A, Z, Y) \psi^*_\nu (dA, dZ, dY) dG (\nu) = p_M K.
$$

And product markets clear by

$$
Y p_S^{-\sigma} = S \quad \text{and} \quad Y p_M^{-\sigma} + \delta K = M,
$$

given the conditional factor demand implied by the final good technology.

5.4 Inter-sector Earnings Differentials

In the stationary equilibrium the service sector contains a higher proportion of taste driven entrepreneurs. The fixed cost in the manufacturing sector raises the minimum efficient scale of manufacturing firm making it more attractive to higher ability households. In a calibrated version of the model, median earnings differentials between occupations are more pronounced for the service sector.

Table 7 reports differences in median earnings for the manufacturing and non manufacturing sectors estimated from the 1996 and 2001 panels of the SIPP.\(^{18}\) The pattern from the two sector model is consistent with the estimates from the SIPP where gaps in the manufacturing sector are roughly half the size of the non manufacturing sector. This is only true for small firms. In the model, since household ability applies equally to both services and manufacturing, higher ability households would all sort into manufacturing. In practice, higher ability households run firms in both sectors. This could be accommodated by introducing sector specific abilities, but I leave that as a separate task. The primary goal of the two sector extension is to check the whether tastes that do not scale with the business are a reasonable assumption.

\(^{18}\)The 2004 panel changes the industry coding system, making cross panel comparisons impossible.
6 Conclusion

I find large and persistent gaps between the earnings of business owners and the earnings of comparable workers. By decomposing the distribution of earnings, I find these differences are driven almost entirely by the relatively low earnings of small business owners. Even after ten years, the gaps between employment and business earnings only narrow slightly for this group. These gaps are problematic for standard models of firm heterogeneity with an occupational choice where households select on comparative advantage. These models have difficulty explaining the existence of small scale lower performing businesses without credit constraints or transient bad shocks. The implication of both alternative explanations is that these lower performing firms are only temporarily small. They will save their way into an efficient scale or exit. This is at odds with the persistence I observe in the data.

I develop a model of firm dynamics that gives preference heterogeneity a first order effect in the occupational choice decision. I find that when calibrated to generate the same gaps in earnings, that over 40 percent of businesses would exit if not for their tastes for entrepreneurship. The implied value of the non pecuniary compensation for this group is large and roughly equal to average earnings as employees. The implication is that it may not be innocuous to abstract from preference heterogeneity when studying aggregate firm dynamics, especially as it relates to small businesses. In sectors with higher entry costs, such as manufacturing, they are less of a confounding force.

References


Table 1: Description of the SIPP Sample

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Business</td>
<td>Owners</td>
<td>Business</td>
<td>Owners</td>
<td>Business</td>
<td>Owners</td>
</tr>
<tr>
<td>Age</td>
<td>39.3</td>
<td>44.7</td>
<td>40.0</td>
<td>45.6</td>
<td>40.6</td>
<td>45.4</td>
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<tr>
<td>Married</td>
<td>65.9</td>
<td>78.0</td>
<td>63.6</td>
<td>76.2</td>
<td>61.9</td>
<td>73.7</td>
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<td>Black</td>
<td>10.1</td>
<td>4.7</td>
<td>10.1</td>
<td>4.1</td>
<td>9.8</td>
<td>3.9</td>
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<tr>
<td>Disability</td>
<td>3.7</td>
<td>3.8</td>
<td>3.1</td>
<td>3.7</td>
<td>3.9</td>
<td>4.2</td>
</tr>
<tr>
<td>Retired</td>
<td>4.7</td>
<td>5.8</td>
<td>4.5</td>
<td>5.5</td>
<td>3.5</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Education
- Less than HS: 12.3, 9.6, 12.5, 9.2, 8.9, 6.2
- High School: 33.0, 30.5, 32.1, 27.1, 29.6, 25.6
- Some College: 28.7, 26.1, 27.9, 27.4, 33.6, 33.0
- Bachelor’s Degree: 16.9, 19.7, 18.0, 21.4, 18.2, 22.4
- Graduate Degree: 9.0, 14.0, 9.5, 14.8, 9.7, 12.8

Tenure
- Weekly Hours: 44.5, 51.4, 43.9, 49.7, 43.7, 49.1
- Percent of sample: 88.2, 11.8, 88.4, 11.6, 89.8, 10.2
- Annual observations: 49,757, 6,826, 32,001, 4,316, 40,626, 4,902

Notes: Each panel contains from 3 to 4 years of annual observations. Each year, an individual is a business owner if he reported owning the same business for more than 12 weeks, working at least 35 hours per week, and has or expects at least $2500 in annual sales. An individual is a worker if he worked at the same firm for more than 12 weeks and works at least 35 hours per week. Individuals who work and own businesses are assigned to the type with the most hours. A full description of the sample construction is in the data appendix. All results are weighted.

Table 2: Distribution of Firms in SIPP and Overall

<table>
<thead>
<tr>
<th>Percent</th>
<th>Firm Age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt; 3 Years</td>
</tr>
<tr>
<td>Business Owners</td>
<td>95.0</td>
</tr>
<tr>
<td>All Firms</td>
<td>88.1</td>
</tr>
</tbody>
</table>

Notes: Business Owners measured in the pooled 1996, 2001, and 2004 panels of the SIPP that span calendar years 1996 to 2008. Small Businesses in the SIPP are defined as having fewer than 25 employees. The distribution of all firms is from the year 2005 and measured in the Census Bureau Business Dynamic Statistics Release (BDS). Small businesses in the BDS are enterprises with fewer than 20 employees over all operating establishments. All survey results are weighted.
Table 3: Differences in Weekly Earnings between Business Owners and Workers in 2008 Dollars

<table>
<thead>
<tr>
<th>Percentile</th>
<th>25th</th>
<th>Median</th>
<th>75th</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. In Business fewer than 10 Years</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small Business</td>
<td>-332.05</td>
<td>-239.56</td>
<td>-90.30</td>
<td>-125.35</td>
</tr>
<tr>
<td></td>
<td>(4.76)</td>
<td>(5.86)</td>
<td>(7.89)</td>
<td>(15.28)</td>
</tr>
<tr>
<td>Larger Business</td>
<td>64.54</td>
<td>294.88</td>
<td>1025.11</td>
<td>992.36</td>
</tr>
<tr>
<td></td>
<td>(22.34)</td>
<td>(29.49)</td>
<td>(41.39)</td>
<td>(151.87)</td>
</tr>
<tr>
<td><strong>B. In Business at least 10 Years</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small Business</td>
<td>-334.93</td>
<td>-235.23</td>
<td>-76.36</td>
<td>-88.21</td>
</tr>
<tr>
<td></td>
<td>(5.26)</td>
<td>(6.42)</td>
<td>(8.49)</td>
<td>(18.19)</td>
</tr>
<tr>
<td>Larger Business</td>
<td>144.81</td>
<td>493.72</td>
<td>2586.72</td>
<td>1493.33</td>
</tr>
<tr>
<td></td>
<td>(18.71)</td>
<td>(22.76)</td>
<td>(30.92)</td>
<td>(139.65)</td>
</tr>
</tbody>
</table>

| Time Effects       | Yes | Yes | Yes | Yes | Yes |
| Demographics       | Yes | Yes | Yes | Yes | Yes |
| Education and Experience | Yes | Yes | Yes | Yes | Yes |
| Individual Fixed Effects | Yes |        |       |       |       |

| R²            | 0.11    | 0.15    | 0.19    | 0.21    | 0.19    |

Notes: Table summarizes regression output from OLS and quantile regressions of average weekly earnings on controls and dummies for business earnings and business types. Robust standard errors are in parentheses. Earnings are deflated using annual averages of the CPI-UW. $R^2$ indicates pseudo $R^2$ for quantile regressions, and overall $R^2$ for fixed effect estimation (within variation was 53 percent of overall). Demographic controls include race and marital status. Additional controls include work limiting disability, retirement status, education dummies, and potential labor market experience and its square. All regression results are weighted.

Table 4: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline</th>
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<tbody>
<tr>
<td>Collateral Constraint</td>
<td>$\lambda$</td>
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<tr>
<td>Entrepreneurial Tastes</td>
<td>$P{\nu = 0.5}$</td>
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<tr>
<td>Capital Intensity</td>
<td>$\alpha$</td>
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<tr>
<td>Span of Control</td>
<td>$\eta$</td>
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<tr>
<td>Depreciation Rate</td>
<td>$\delta$</td>
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<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
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<tr>
<td>Persistence of log Z</td>
<td>$\rho_z$</td>
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<td>Volatility of log Z</td>
<td>$\sigma_z$</td>
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Table 5: Comparison of Calibrated Model with Preference Heterogeneity against Alternative Sources of Heterogeneity

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
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<tbody>
<tr>
<td>Business Ownership Rate</td>
<td>11.6</td>
<td>12.7</td>
<td>7.4</td>
<td>13.6</td>
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<tr>
<td>Percent marginal</td>
<td>48.3</td>
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<tr>
<td>Non pecuniary compensation</td>
<td>1.14</td>
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<tr>
<td>Ratio of Business Earnings to Employment Earnings</td>
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<tr>
<td>Medians</td>
<td>0.7</td>
<td>0.68</td>
<td>2.36</td>
<td>2.46</td>
</tr>
<tr>
<td>Averages</td>
<td>0.94</td>
<td>1.56</td>
<td>4.35</td>
<td>4.85</td>
</tr>
<tr>
<td>Conterfactual: Median Ratio of Business to Potential Employment Earnings</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>1.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu &gt; 0$</td>
<td>0.49</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu = 0$</td>
<td>2.58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm Distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent of Empl in</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 20 percent</td>
<td>89.2</td>
<td>80.9</td>
<td>77</td>
<td>79.2</td>
</tr>
<tr>
<td>Top 10 percent</td>
<td>82.2</td>
<td>60</td>
<td>53.7</td>
<td>56.8</td>
</tr>
<tr>
<td>Percent Entering</td>
<td>5.2</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Percent Exiting</td>
<td>41.6</td>
<td>21.4</td>
<td>24.8</td>
<td>17.4</td>
</tr>
</tbody>
</table>

Notes: Table compares the benchmark calibration of the model with preference heterogeneity against two alternatives. Model 2 eliminates preference heterogeneity and Model 3 eliminates preference heterogeneity and tightens credit constraints. Data on earnings differences, entry and exit, and business ownership rate are measured using pooled SIPP panels from 1996, 2001, and 2004. Data on the distribution of firms are from the 2008 Statistics of US Businesses and exclude non employer firms.
Table 6: Preference Heterogeneity and the Wealth Distribution

<table>
<thead>
<tr>
<th>Percent of Wealth Distribution Held by Businesses</th>
<th>Data</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>41.6</td>
<td>38.2</td>
<td>30.7</td>
<td>56.4</td>
</tr>
<tr>
<td>Top 20</td>
<td>42</td>
<td>49.1</td>
<td>39.8</td>
<td>68</td>
</tr>
<tr>
<td>Top 10</td>
<td>30</td>
<td>59.1</td>
<td>47.3</td>
<td>74.2</td>
</tr>
<tr>
<td>Ratio of Median Business Wealth to Overall Wealth</td>
<td>5.7</td>
<td>2.6</td>
<td>2.8</td>
<td>5.23</td>
</tr>
<tr>
<td>Percent of Wealth Held by</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 20</td>
<td>81</td>
<td>71.1</td>
<td>73.2</td>
<td>79.5</td>
</tr>
<tr>
<td>Top 10</td>
<td>67</td>
<td>55.8</td>
<td>59.5</td>
<td>67.4</td>
</tr>
<tr>
<td>Gini</td>
<td>0.8</td>
<td>0.58</td>
<td>0.56</td>
<td>0.52</td>
</tr>
<tr>
<td>Debt to Output Ratio</td>
<td>2.3</td>
<td>1.92</td>
<td>1.75</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Notes: Model 1 is the benchmark calibration with preference heterogeneity. Model 2 eliminates preference heterogeneity and Model 3 eliminates preference heterogeneity and tightens credit constraints. Data on the distribution of wealth are from Cagetti and De Nardi (2005). Data on the debt to total output ratio are from Midrigan and Xu (2010).
Table 7: Differences in Weekly Earnings between Business Owners and Workers in 2008 Dollars for Manufacturing and Non Manufacturing Sectors

<table>
<thead>
<tr>
<th>Percentile</th>
<th>25th</th>
<th>Median</th>
<th>75th</th>
<th>Average</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non Manufacturing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. In Business fewer than 10 Years</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small Business</td>
<td>-329.47</td>
<td>-236.07</td>
<td>-90.31</td>
<td>-125.79</td>
<td>-147.13</td>
</tr>
<tr>
<td>(5.05)</td>
<td>(5.88)</td>
<td>(8.48)</td>
<td>(15.65)</td>
<td>(10.32)</td>
<td></td>
</tr>
<tr>
<td>Larger Business</td>
<td>78.32</td>
<td>298.19</td>
<td>1189.72</td>
<td>1023.74</td>
<td>558.78</td>
</tr>
<tr>
<td>(23.92)</td>
<td>(30.06)</td>
<td>(45.37)</td>
<td>(159.66)</td>
<td>(44.34)</td>
<td></td>
</tr>
<tr>
<td>B. In Business at least 10 Years</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small Business</td>
<td>-332.54</td>
<td>-231.43</td>
<td>-75.13</td>
<td>-83.84</td>
<td>-51.34</td>
</tr>
<tr>
<td>(5.56)</td>
<td>(6.43)</td>
<td>(9.10)</td>
<td>(18.71)</td>
<td>(12.48)</td>
<td></td>
</tr>
<tr>
<td>Larger Business</td>
<td>146.06</td>
<td>528.38</td>
<td>2603.57</td>
<td>1497.83</td>
<td>767.10</td>
</tr>
<tr>
<td>(20.07)</td>
<td>(23.14)</td>
<td>(33.63)</td>
<td>(145.56)</td>
<td>(41.07)</td>
<td></td>
</tr>
<tr>
<td><strong>Manufacturing</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. In Business fewer than 10 Years</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small Business</td>
<td>-163.23</td>
<td>-125.58</td>
<td>169.87</td>
<td>122.59</td>
<td>48.82</td>
</tr>
<tr>
<td>(21.24)</td>
<td>(25.48)</td>
<td>(37.34)</td>
<td>(73.81)</td>
<td>(52.23)</td>
<td></td>
</tr>
<tr>
<td>Larger Business</td>
<td>-79.31</td>
<td>167.25</td>
<td>741.77</td>
<td>651.61</td>
<td>649.16</td>
</tr>
<tr>
<td>(70.95)</td>
<td>(90.53)</td>
<td>(131.60)</td>
<td>(405.07)</td>
<td>(150.93)</td>
<td></td>
</tr>
<tr>
<td>B. In Business at least 10 Years</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small Business</td>
<td>-133.17</td>
<td>-101.90</td>
<td>50.24</td>
<td>59.53</td>
<td>56.76</td>
</tr>
<tr>
<td>(24.25)</td>
<td>(29.02)</td>
<td>(41.21)</td>
<td>(73.56)</td>
<td>(64.33)</td>
<td></td>
</tr>
<tr>
<td>Larger Business</td>
<td>244.64</td>
<td>430.71</td>
<td>1916.24</td>
<td>1564.81</td>
<td>1004.29</td>
</tr>
<tr>
<td>(63.01)</td>
<td>(75.08)</td>
<td>(102.31)</td>
<td>(488.49)</td>
<td>(145.69)</td>
<td></td>
</tr>
</tbody>
</table>

**Time Effects** | Yes | Yes | Yes | Yes | Yes |
**Demographic** | Yes | Yes | Yes | Yes | Yes |
**Education and Experience Controls** | Yes | Yes | Yes | Yes | Yes |
**Individual Fixed Effects** | Yes | | | | |

**Observations** | 138428 | 138428 | 138428 | 138428 | 138428 |
**R2** | 0.12 | 0.15 | 0.19 | 0.21 | 0.19 |

Notes: Table summarizes regression output from OLS and quantile regressions of average weekly earnings on controls and dummies for business earnings and business types. Robust standard errors are in parentheses. Earnings are deflated using annual averages of the CPI-UW. $R^2$ indicates pseudo $R^2$ for quantile regressions, and overall $R^2$ for fixed effect estimation (within variation was 53 percent of overall). Demographic controls include race and marital status. Additional controls include work limiting disability, retirement status, education dummies, and potential labor market experience and its square. All regression results are weighted.
Figure 1: Conditional distribution of weekly earnings re-centered on unconditional sample mean.

Figure 2: Conditional distribution of weekly earnings re-centered on unconditional sample mean.
Household $i$ enters period $t$ with $A_{it}$, $Z_{it}$, $Y_{it}$

Work for a business

Run a business

Earn: $wY_{it} + rA_{it}$

(1) Borrow costs:

$$wL_{it} + K_{it}$$

(2) Earn:

$$Z_{it}^{1-\eta} (K_{it}^{\theta} L_{it}^{1-\theta})^{\eta} + rA_{it} + (1-\delta)K_{it}$$

(3) Repay debt:

$$(wL_{it} + K_{it})(1+r')$$

Consume and save

Household $i$ ends period $t$ with $A_{it+1}$

Figure 3: Within period timing from $t$ to $t + 1$

Figure 4: Within period timing from $t$ to $t + 1
Figure 5: Earnings Differences for Manufacturing and Services Sectors
A Omitted Proofs

A.1 Credit Constrained Solution

The entrepreneur maximizes

$$Z^{1-\eta} \left( K^\alpha L^{1-\alpha} \right)^\eta - w (1 + r) L - (r + \delta) K$$

subject to the credit constraint

$$wL + K \leq \lambda A.$$ 

Letting $\mu$ be the multiplier on the constraint. Then the Kuhn-Tucker conditions are

$$K : \eta Z^{1-\eta} \left( K^\alpha L^{1-\alpha} \right)^{\eta-1} \alpha K^{\alpha-1} L^{1-\alpha} = r + \delta + \mu$$

and

$$L : \eta Z^{1-\eta} \left( K^\alpha L^{1-\alpha} \right)^{\eta-1} (1 - \alpha) K^\alpha L^{-\alpha} = w \left( 1 + r + \mu \right).$$

I label the optimal capital to labor ratio $\kappa = \frac{\alpha}{1-\alpha} \frac{w(1+r+\mu)}{r+\delta+\mu}$ After some algebra you can show

$$K = Z \left( \frac{\alpha}{r+\delta+\mu} \eta \kappa^{\eta(1-\alpha)} \right)^{\frac{1}{1-\eta}}$$

and

$$L = Z \left( \frac{1 - \alpha}{w(1 + r + \mu)} \eta \kappa^{\eta(1-\alpha)} \right)^{\frac{1}{1-\eta}}$$

so then

$$w \left( \frac{1 - \alpha}{w(1 + r + \mu)} \eta \kappa^{\eta(1-\alpha)} \right)^{\frac{1}{1-\eta}} + \left( \frac{\alpha}{r+\delta+\mu} \eta \kappa^{\eta(1-\alpha)} \right)^{\frac{1}{1-\eta}} \leq \lambda \frac{A}{Z}.$$ 

Suppose $\mu > 0$ so that the constraint is binding, after substituting in for $\kappa$, the multiplier $\mu$ is defined implicitly by

$$\eta^{\frac{1}{1-\eta}} \left( \frac{\alpha}{1-\alpha} \frac{w(1+r+\mu)}{r+\delta+\mu} \right)^{\frac{\alpha}{1-\eta}} \left( \frac{1 - \alpha}{w(1 + r + \mu)} \right)^{\frac{1}{1-\eta}} \left( w + \frac{w(1 + r + \mu)}{r+\delta+\mu} \frac{\alpha}{1-\alpha} \right) = \lambda \frac{A}{Z}.$$ 

Given fixed $\delta$, $r$, $w$, $\alpha$, and $\lambda$, the multiplier depends only on the ratio $b = A/Z$. Unfortunately, this cannot be solved analytically for $\mu$, but it is straightforward to solve numerically.

I label as $b^*$ the lower threshold for $A/Z$ at which the collateral constraint starts to bind,
i.e., when $\mu = 0$

$$b^* = \frac{1}{\lambda} \eta^{1-\eta} \left( \frac{\alpha}{1 - \alpha - r + \delta} \right)^{\eta \alpha} \left( \frac{1 - \alpha}{W (1 + r)} \right)^{\frac{1}{1-\eta}} \left( W + \frac{W (1 + r) \alpha}{r + \delta (1 - \alpha)} \right).$$

### B Perfect Credit Benchmark

When $\lambda \to \infty$ the firm’s problem no longer depends on assets $A$ and all firms immediately adjust to their efficient scale. From above then

$$K = \frac{\alpha}{r + \delta} \eta \left( \frac{\alpha}{1 - \alpha - r + \delta} \right)^{-\eta (1 - \alpha)} \right)^{\frac{1}{1-\eta}}$$

and

$$L = \frac{1 - \alpha}{w (1 + r)} \eta \left( \frac{\alpha}{1 - \alpha - r + \delta} \right)^{\eta \alpha} \right)^{\frac{1}{1-\eta}}$$

since we assume that $Z$ follows

$$\log Z_{t+1} - \mu_z = \rho (\log Z_t - \mu_z) + \sigma_z \varepsilon_{t+1}$$

with $\varepsilon_{t+1} \sim N(0, 1)$ and supposing firms produce for any $Z > 0$ then the stationary distribution of $Z$ will be

$$\log Z \sim N \left( \mu_z, \frac{\sigma_z^2}{1 - \rho^2} \right)$$

so that $E[Z] = \exp \left( \mu_z + \frac{1}{2} \frac{\sigma_z^2}{1 - \rho^2} \right)$ and $\text{Var}[Z] = \left( \exp \left( \frac{\sigma_z^2}{1 - \rho^2} - 1 \right) \right) \exp \left( 2 \mu_z + \frac{\sigma_z^2}{1 - \rho^2} \right)$, and

$$E[K] = \exp \left( \mu_z + \log \tilde{K} + \frac{1}{2} \frac{\sigma_z^2}{1 - \rho^2} \right)$$

for

$$\tilde{K} = \left( \frac{\alpha}{r + \delta} \eta \left( \frac{\alpha}{1 - \alpha - r + \delta} \right)^{-\eta (1 - \alpha)} \right)^{\frac{1}{1-\eta}}$$

Try calibrating this distribution for fixed $\alpha = 0.33, \delta = 0.06$, $w = 1$, and $r = 0.04$ by choosing $\eta$ and $\rho$ and $\sigma_z$ and $\mu_z$. Normalize $\mu_z = -\frac{1}{2} \frac{\sigma_z^2}{1 - \rho^2}$ so that $E[Z] = 1.$
C Numerical Solution Algorithm

I solve the model numerically using the following algorithm. I go into some detail, because I use a version of the endogenous grid method that is suitably modified to address the non concavities in the value function introduced by the discrete occupational choice. The method is based on Fella (2011).

State Space

1. Choose a grid for assets $A = \{a_1, a_2, \ldots, a_{N_a}\}$ of length $N_a$. I use an exponentially spaced grid to provide more nodes over the region where the value function has the most curvature.

2. Approximate the Markov process for $Z_t$ using an $N_z$ state Markov chain using the Tauchen (1986) method. Let the values for the Markov state be $Z = \{z_1, z_2, \ldots, z_{N_z}\}$ and the transition matrix be $P_z$.

3. Model the $Y_t$ process as an $N_y$ state regular Markov chain with values $Y = \{y_1, \ldots, y_{N_y}\}$ and transition matrix $P_y$.

4. Let the Markov state for the skill vector be denoted $s \in S = Y \times Z$ with transition matrix $P_s = P_y \otimes P_z$, so that $s_1$ is values of $y$ and $s_2$ is values of $z$.

Credit Constrained Profit Maximization Solve the profit maximization problem of (1) subject to the collateral constraint (2) for each value of $a \in A$ and $z \in Z$. I label this $\Pi(a,s_2)$.

Decision Problem For each household type $v \in \{\nu_1, \ldots, \nu_V\}$ I solve the households decision problem using a version of the method of endogenous grid points described by Carroll (2006) and Barillas and Fernandez-Villaverde (2007).

1. Define a new set of Bellman equations equivalent to (3) to (5) in terms of market resources $X$

$$V_k(a, s) = \max_{j \in \{W, B\}} \left\{ \tilde{V}_k^W \left( \left(1 + r_f\right) a + w s_1, s \right) \tilde{V}_k^B \left( \left(1 + r_f\right) a + \Pi(a, s_2), s \right) \right\}$$

with

$$\tilde{V}_k^W(x, s) = \max_{a' \geq a_1} \left\{ \frac{(x - a')^{1-\gamma}}{1 - \gamma} + \beta E[V_k(a', s') | s] \right\}$$
and

\[ \tilde{V}_k^B (x, s) = \max_{a' \geq a_1} \left\{ \frac{(x - a')^{1-\gamma}}{1 - \gamma} + \nu_k + \beta E[V_k (a', s') | s]\right\}. \]

Letting \( f (a', s) = \beta E[V_k (a', s') | s] \), then both equations share the same first order condition determining savings

\[ (x - a')^{-\gamma} \geq f_a (a', s), \]

which holds with equality when \( a' > a_1 \).

2. The method of endogenous gridpoints will fix the values of \( a' \) (instead of \( a \)) on the set \( \mathcal{A} \) and then solve for values of \( x \) where the choice \( a' \) is optimal. I define sets \( \mathcal{X}^W (\mathcal{A}, S) = \{(1 + r) A + S_1 w\} \) and \( \mathcal{X}^B (\mathcal{A}, S) = \{(1 + r) A + \Pi (A, S_2)\} \) that represent possible values of market resources \( x \) corresponding to entering a period with \( a \in \mathcal{A} \) and skills \( s \in \mathcal{S} \).

3. Since this needs to be solved for each \( k \), I suppress the \( k \) from the notation. For a type \( k \), make an initial guess \( f^0 (a', s) \) for \( f (a', s) \) for values \( a' \in \mathcal{A} \) and \( s \in \mathcal{S} \). I use the expected discounted value of consuming maximal permanent income \( c (a, s) = ra + \max\{wS_1, \Pi (a, S_2)\} \). Let \( V^0 \) represent the value of this policy then

\[ f^0 (a', s) = \beta E[V^0 (a', s') | s] \]

I represent \( f \) at the nodes as a \( N_a \times N_s \) matrix so that

\[ f^0 (a', s) = \frac{(rA + \max\{wS_1, \Pi (A, S_2)\})^{1-\gamma}}{1 - \gamma} \left( I - \beta P_s \right)^{-1} \]

with \( A, S_1 \), and \( S_2 \) being \( N_a \times N_s \) mesh of values.

4. Numerically differentiate \( f^0 (a', s) \) to determine \( f_a^0 (a', s) \).

5. Find the consumption consistent with next period \( a' \in \mathcal{A} \) using the first order condition

\[ \tilde{c}_{i+1} (a', s) = \left( f_a^i (a', s) \right)^{1-\gamma}. \]

6. The

7. Find the value of today’s \( x \) consistent with choosing \( a' \)

\[ \tilde{x}_{i+1} (a', s) = a' + \tilde{c}_{i+1} (a', s). \]
8. Next I compute $\tilde{V}^W$ and $\tilde{V}^B$ on sets $\mathcal{X}^W$ and $\mathcal{X}^B$ respectively. To do this I first find the values at the $\tilde{x}$ from the previous step

$$
\tilde{V}_{W,i+1}^W(\tilde{x}(a',s), s) = \frac{(\tilde{x} - a')^{1-\gamma}}{1 - \gamma} + f_i(a', s)
$$

and

$$
\tilde{V}_{B,i+1}^B(\tilde{x}(a',s), s) = \frac{(\tilde{x} - a')^{1-\gamma}}{1 - \gamma} + \nu + f_i(a', s)
$$

and then linearly interpolate on the sets $\mathcal{X}^W$ and $\mathcal{X}^B$ to find $\hat{\tilde{V}}_{W,i+1}^W$ and $\hat{\tilde{V}}_{B,i+1}^B$.

9. Then I update the value of $f$ by first solving the occupational choice problem

$$
V(a,s) = \max_{j \in \{W,B\}} \left\{ \tilde{V}_{W,i+1}^W, \tilde{V}_{B,i+1}^B \right\}
$$

on the set $\mathcal{A} \times \mathcal{S}$, which corresponds to $\mathcal{X}^W$ and $\mathcal{X}^B$. I let $j(a,s)$ be the optimal occupational choice. Then I can update $f$ by computing the discounted expectation

$$
f_{i+1}^{i+1}(a',s) = \beta E[V(a',s')|s].
$$

10. I update the derivative $f_{a,i+1}^{i+1}$. In practice, I had difficulty using a numerical derivative, so I find this derivative using an envelope condition. I find $c(x,s)$ by interpolating on the sets $\mathcal{X}^W$ and $\mathcal{X}^B$ and then find

$$
\frac{\partial \hat{V}}{\partial a}(a,s) = \left(c^W(x,s)\right)^{-\gamma} (1 + r^f) 1_{\{j=W\}} + \left(c^B(x,s)\right)^{-\gamma} (1 + r^f + \Pi_a(a,s)) 1_{\{j=B\}}
$$

and then finding

$$
f_{a,i+1}^{i+1}(a',s) = \beta E \left[ \frac{\partial \hat{V}_{i+1}^{i+1}}{\partial a}(a',s') \right].
$$

11. For each $k$, repeat steps 3 to 9 until $\frac{\|f_{a,i+1}^{i+1} - f_{a,i+1}^i\|}{\|f_{a,i+1}^i\|}$ is sufficiently small.

12. I recover $a^{-1}(a',s)$, today’s assets as a function of next period’s assets on the grid, by solving for each $(a',s) \in \mathcal{A} \times \mathcal{S}$

$$
a^W = \frac{a' + \tilde{c}(a',s) - ws_1}{1 + r^f}
$$

19 Description of the procedure used to deal with the corner solutions
and
\[ a^B (1 + r^f) + \Pi (a^B, s) = a' + \tilde{c} (a', s) . \]

Then
\[ a^{-1} (a', s) = a^W (a', s) \mathbf{1}_{\{ j(a^W, s) = W \}} + a^B (a', s) \mathbf{1}_{\{ j(a^B, s) = B \}} , \]

where \( \hat{j} (a) \) is the interpolated occupational choice on the recovered values of today’s assets.

**Stationary Measures** To find the aggregate stationary distribution I first characterize the stationary measures for each type \( k \) and then aggregate them weighting by \( g (\nu) \).

1. For each type \( k \), pick an initial measure \( \psi^0_k (a, s) \) defined over \( \mathcal{A} \) and \( \mathcal{S} \). I use uniformly distributed assets and \( s \) initialized at its stationary distribution.

2. To compute \( \psi^{i+1}_k \) for each \( (a, s) \in \mathcal{A} \times \mathcal{S} \) I use the inverse policy function recovered from the solution to the decision problem

\[ \psi^{i+1} (a', s') = \sum_{s' \in \mathcal{S}} \hat{\psi}^i (a^{-1} (a', s), s) F (s'|s) , \]

where \( \hat{\psi}^i \) is the value of the measure interpolated on the set of \( a = a'^{-1} (a', s) \) values.

3. Repeat step 2 until \( \frac{\| \psi^{i+1} - \psi^i \|}{\| \psi^{i+1} \|} \) is sufficiently small.

4. With \( \psi_k \) for each \( k \) then compute

\[ \Psi (a', s') = \sum \psi_v (a', s') g (\nu) . \]

**Market Clearing** The prices \( r^f \) and \( w \) must clear the labor and asset markets. I iterate on pairs \( (r^f, w) \) until

\[ \sum_s \sum_a \left( K (a, s) \mathbf{1}_{\{ j(a, s) = B \}} - a \right) \Psi (a, s) = 0 \]

and

\[ \sum_s \sum_a \left( N (a, s) \mathbf{1}_{\{ j(a, s) = B \}} - s_2 \mathbf{1}_{\{ j(a, s) = W \}} \right) \Psi (a, s) = 0 . \]

For each iteration, I solve the profit maximization problem, decision rules, and stationary measure for the values of \( r^f \) and \( w \).